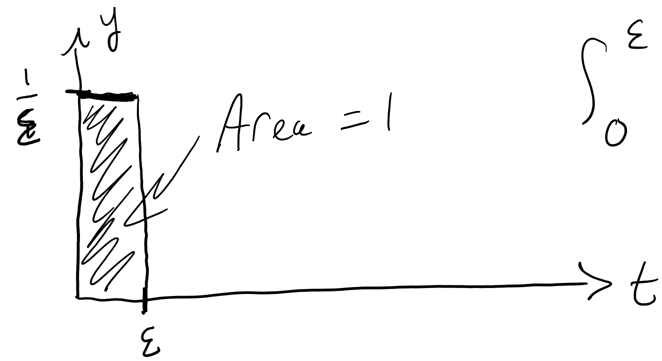
Section 5.7 Delta Function and Impulse Response

Friday, May 1, 2020 9:13 AM

Define the delta function $\delta(t)$ such that

$$\int_a^a \delta(t)\,dt = 1$$

Impulse function $P_\varepsilon(t)$



$$\int_0^\varepsilon P_\varepsilon(t) = 1$$

Impulse force of size F with short duration $F\delta(t)$

1. $$\int_a^b f(t)\,\delta(t - t_0)\,dt = \begin{cases} f(t_0) & \text{if } a \le t_0 \le b \\ 0 & \text{otherwise} \end{cases}$$

2. $$\mathcal{L}\{\delta(t - t_0)\} = e^{-t_0 s}$$ row #27 on our table

$-2 \le t_0 = 2 \le 5$

$$+ e^{-t})\,\delta(t-2)\,dt = \boxed{1 - e^{-2}}$$

Ex' $$\int^5 (1 + e^{-t})\,\delta(t-2)\,dt = \boxed{0}$$

Ex! $\int_{3}^{5} (1+e^{-t})\,\delta(t-2)\,dt = \boxed{0}$

Ex! $\mathcal{L}\{ y'' + 2y' + 2y = \delta(t-1) \}$ $t_0 = 1$

$y(0)=0, \quad y'(0)=0$

$\mathcal{L}\{y''\} = s^2 Y(s) - s\,y(0) - y'(0)$

$\mathcal{L}\{y'\} = s\,Y(s) - y(0)$

$\mathcal{L}\{\delta(\underbrace{t - t_0}) \} = e^{\underbrace{-t_0 s}}$

$s^2 Y(s) + 2s\,Y(s) + 2Y(s) = e^{-s}$

$$Y(s) = \frac{e^{-s}}{s^2+2s+2} = \frac{e^{-s}}{s^2+2s+1 \;\; +2-1}$$

$$Y(s) = \frac{e^{-s}}{(s+1)^2+1} = e^{-s} \underbrace{\left(\frac{1}{(s+1)^2+1}\right)}_{F(s)}$$

$\alpha = 1$

12. $e^{\alpha t}\sin \omega t$	$\dfrac{\omega}{(s-\alpha)^2+\omega^2}$
13. $e^{\alpha t}\cos \omega t$	$\dfrac{s-\alpha}{(s-\alpha)^2+\omega^2}$
14. $f(t-\alpha)h(t-\alpha)$, $(\alpha \geq 0)$, with $\lvert f(t)\rvert \leq Me^{at}$	$e^{-\alpha s}\boxed{F(s)}$

First find $\mathcal{L}^{-1}\{F(s)\} = f(t)$

then shift $f(t-1)\,h(t-1)$

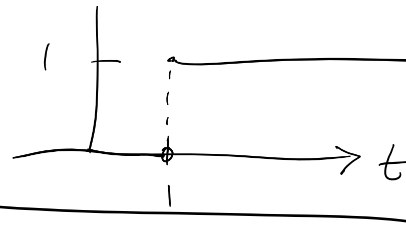
$\mathcal{L}^{-1}\left\{ \frac{1}{\quad} \right\} = e^{-t}\sin t$ row #12

$$\hookrightarrow f(t) = \mathcal{L}^{-1}\left\{ \frac{1}{(s+1)^2+1} \right\} = e^{-t} \sin t \qquad \text{row } \#12$$

$\alpha = -1 \quad \omega = 1$

$$\boxed{y(t) = e^{-(t-1)} \sin(t-1)\; h(t-1)} = \mathcal{L}^{-1}\{Y(s)\} \qquad \text{row } \#14$$

$h(t-1)$



$$\boxed{y(t) = \begin{cases} 0 & t < 1 \\ e^{-(t-1)} \sin(t-1) & t \geq 1 \end{cases}}$$